

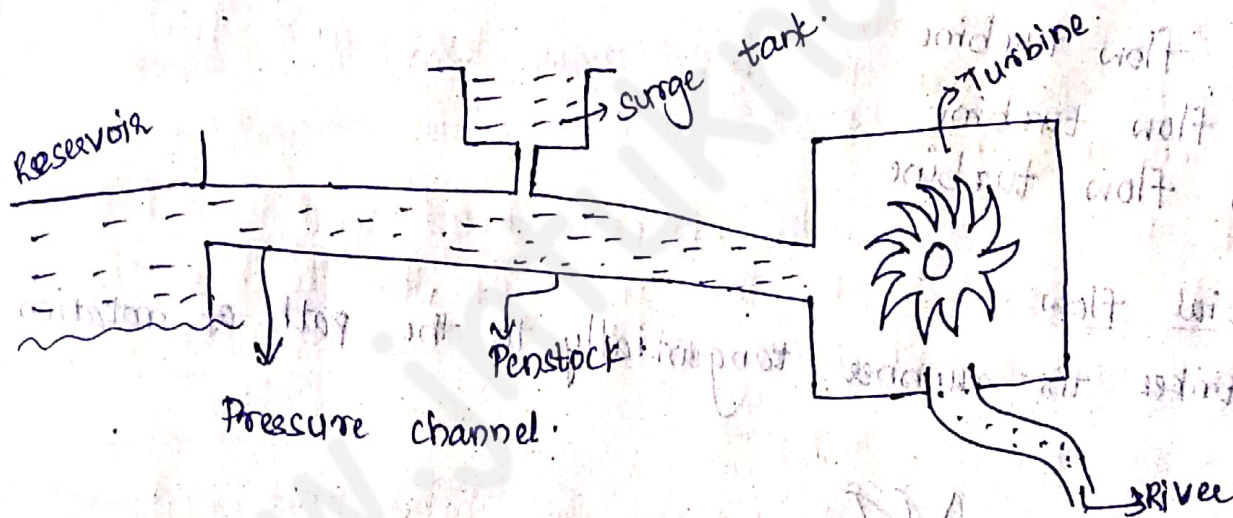
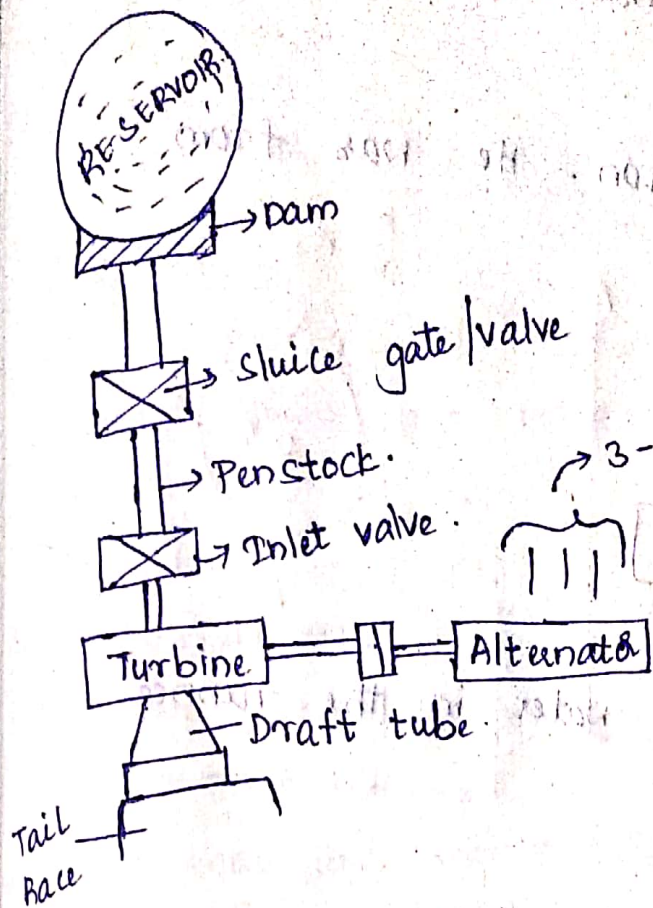
UNIT - V

HYDRAULIC TURBINES

- Layout of a typical hydropower installation -
- Heads and efficiencies.
- classification of turbines
 - ^{***} Pelton wheel
 - ^{***} Francis turbine
 - ^{***} Kaplan turbine

} (P) 14H.
- Working, working proportions, velocity diagram, workdone and efficiency, hydraulic design, draft tube
- Theory and efficiency, Governing of turbines
- Surge tanks
- Unit and specific quantities, selection of turbines, performance characteristics.
- Geometric similarity
- Cavitation.

Flow sheet of hydroelectric power plant:



Classification of turbines:

1. Impulse turbine:

Requires high head and small quantity of flow

Eg: pelton wheel.

2. Reaction turbine:

Requires low head and high rate of flow.

Eg: Francis turbine and Kaplan turbine

Classification of

According to the name of the originator

1. Pelton turbine (or) Pelton wheel

It was discovered by Allen pelton. He was from California USA.

2. Francis turbine

[Bichens Francis]

3. Kaplan turbine [Dr. Victor Kaplan]

According to direction of flow of water in the runner

There are four types of flow.

1. Tangential flow turbine.

2. Radial flow turbine

3. Axial flow turbine

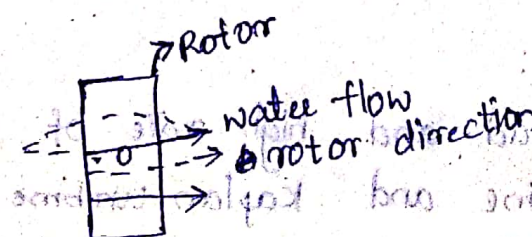
4. Mixed flow turbine

1. Tangential flow:

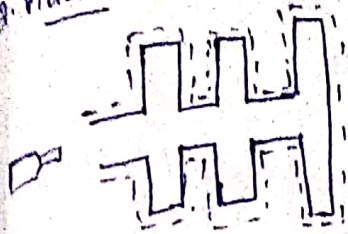
Water strikes the runner tangentially to the path of rotation.



3. Axial flow turbine:



Radial flow



Mixed flow

When the flow is both radial and axial flow then it is known as mixed flow. Water enters with radial flow and leaves with axial flow.

According to water head and quantity of water available:

1. High head and small quantity of water flow.
($H > 250\text{m}$) Pelton.

2. Medium head and medium quantity of water flow.

(60m to 250m) Francis.

3. Low head and large quantity of water flow.
($< 60\text{m}$) Kaplan and propeller turbines.

According to specific speed of the turbine:

$$N_s = \frac{N \sqrt{P}}{H^{5/4}}$$

N = Normal working speed

P = Power output

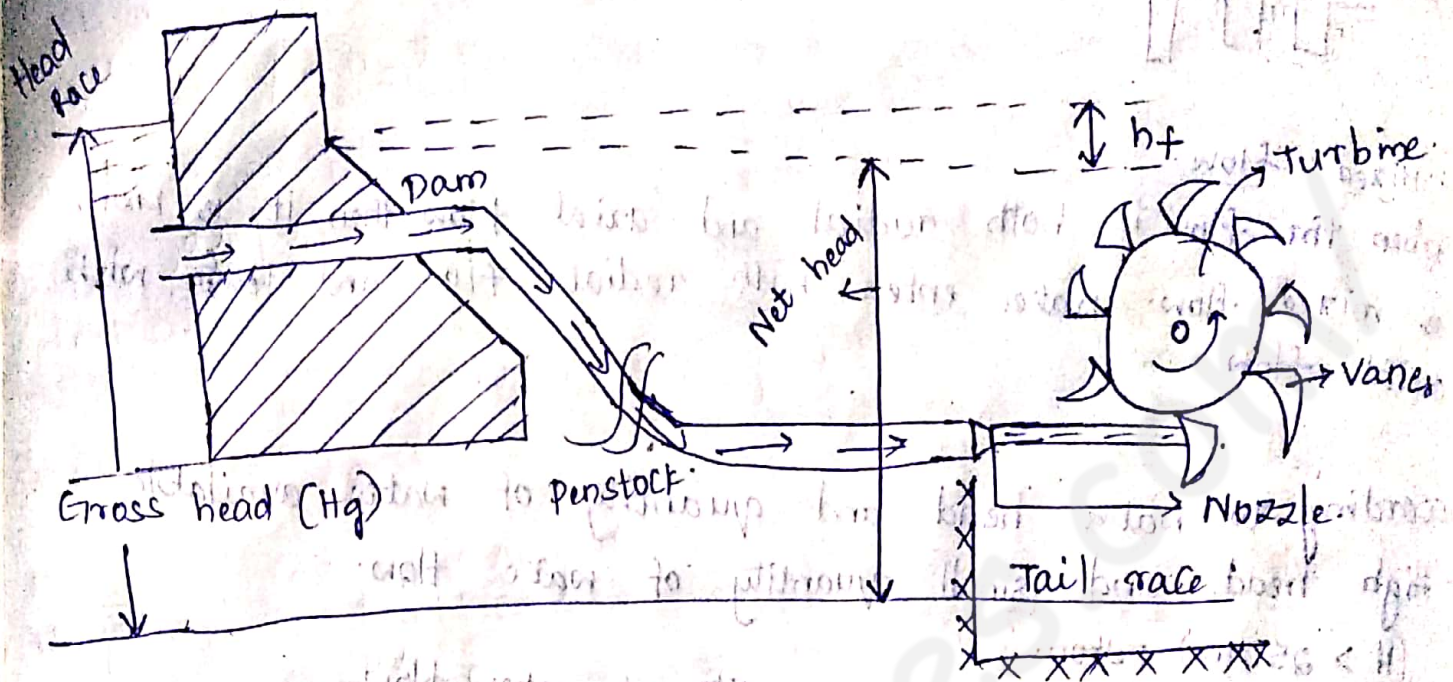
H = The net or effective head in m.

1. Low specific speed: ex: pelton wheel. < 60

2. Medium specific speed (60 to 400) speed ex: Francis

3. High specific speed: > 400 . ex: kaplan turbine.

Heads and efficiencies of a turbine:



Gross head:

The difference between the head race level and tail race level is known as gross head.

Net head: This is also called effective head and is defined as the head available at the inlet of the turbine is known as net or effective head.

$$\therefore \text{Net head } H = H_g - H_f$$

H_f = Total loss of head between the head race and entrance of the turbine.

$$H_f = \frac{4fLV^2}{agD}$$

Efficiency:

Hydraulic efficiency (η_h) = $\frac{\text{Power delivered to runner}}{\text{Power supplied at inlet}}$

$$\eta_h = \frac{R.P}{N.P} = \frac{\text{Rotor power}}{\text{Water power}}$$

$$B.P = \frac{W}{g} \frac{[V_{w1} \pm V_{w2}] \times u}{1000}$$

kW $\rightarrow$ Tangential flow

$$B.P = \frac{W}{g} \frac{[V_{w1} \times u \pm V_{w2} \times u]}{1000}$$

kW $\rightarrow$ Radial flow

Water power

$$W.P = \frac{W \times H}{1000} \text{ kW}$$

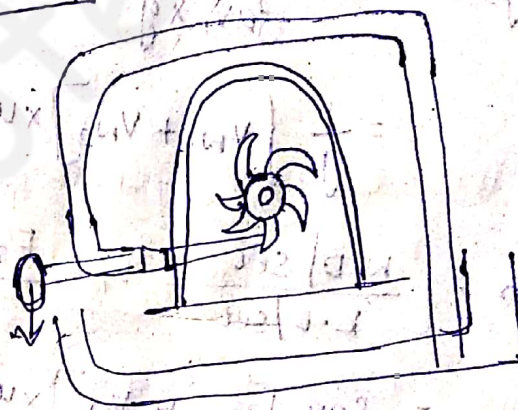
H = Height of water
= $\rho g q$

Strikes the vanes of the turbine per sec

$$W.P = \frac{\rho g Q \times H}{1000}$$

$$= \frac{1000 \times g \times Q \times H}{1000}$$

$$\Rightarrow W.P = g Q H$$



VELOCITY TRIANGLE AND WORKDONE FOR PELTON WHEEL:

$$H = H_g - h_f = \text{Net head.}$$

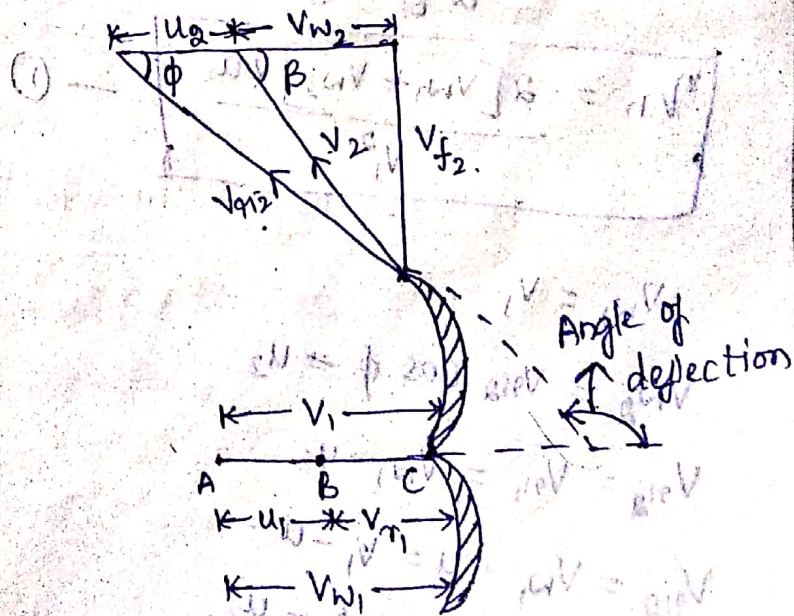
H_g = Gross head

h_f =

$$h_f = \frac{4fLV^2}{\pi g D}$$

D = Diameter of penstock.

$$u_1 = u_2 = u = \frac{\pi D N}{60}$$



$$V_{w1} = V_1$$

$$\cos \phi = \frac{u_2 + V_{w2}}{V_{w1}}$$

$$\Rightarrow V_{w1} \cos \phi - u_2 = V_{w2}$$

$$\theta = \alpha = 0^\circ$$

$$F_x = \rho a V_1 [V_{w1} + V_{w2}]$$

$$\text{Work Done. } W.D = F_x \times u$$

$$= \rho a V_1 [V_{w1} + V_{w2}] \times u$$

$$\text{Weight} = \rho a V_1 \times g$$

$$\frac{W.D}{\text{Weight}} = \frac{\rho a V_1 [V_{w1} + V_{w2}] \times u}{\rho a V_1 \times g}$$

$$= \frac{1}{g} [V_{w1} + V_{w2}] \times u$$

$$\eta_h = \frac{W.D / \text{sec}}{K.E / \text{sec}} = \frac{F_x \times u}{\frac{1}{2} \rho a V_1^2}$$

$$= \frac{\rho a V_1 [V_{w1} + V_{w2}] \times u}{\frac{1}{2} [\rho a V_1] V_1^2}$$

$$\boxed{\eta_h = \frac{2 [V_{w1} + V_{w2}] \times u}{V_1^2}} \quad \text{--- (1)}$$

$$V_{w1} = V_1$$

$$V_{w2} = V_{w1} \cos \phi = u_2$$

$$V_{w1} = V_{w1} = V_{w1} - u_1$$

$$V_{w2} = V_{w1} - u_1 = V_1 - u_1$$

$$V_{w2} = (V_1 - u) \cos \phi = u_2$$

Substitute the ' v_2 ' value in equ ①

$$r_h = \frac{2 \left[v_1 + (v_1 - u) \cos \phi - u \right] \times u}{v_1^2}$$

$$= \frac{2 \left[(v_1 - u) + (v_1 - u) \cos \phi \right] \times u}{v_1^2}$$

$$r_h = \frac{2 (v_1 - u) (1 + \cos \phi) \times u}{v_1^2} \quad \text{--- ②}$$

$$\frac{d}{du} [r_h] = 0$$

$$\frac{d}{du} \left[\frac{2 (v_1 - u) (1 + \cos \phi) \times u}{v_1^2} \right] = 0$$

$$\frac{2 (1 + \cos \phi)}{v_1^2} \frac{d}{du} [v_1 u - u^2] = 0$$

$$\frac{d}{du} [v_1 u - u^2] = 0$$

$$v_1 - 2u = 0$$

$$v_1 = 2u \Rightarrow \therefore u = \frac{v_1}{2}$$

Substitute 'u' value in equ ②

$$r_h = \frac{2 \left(v_1 - \frac{v_1}{2} \right) (1 + \cos \phi) \times \frac{v_1}{2}}{v_1^2}$$

$$= \frac{2 \left(\frac{2v_1 - v_1}{2} \right) (1 + \cos \phi) \times \frac{v_1}{2}}{v_1^2}$$

$$r_{h \max} = \frac{1}{2} (1 + \cos \phi)$$

Efficiency formulas:

1. Mechanical efficiency (η_m) = $\frac{\text{Power at the shaft of a turbine}}{\text{Power delivered by water to the runner}}$
$$= \frac{S.P}{R.P} = \frac{\text{shaft power}}{\text{runner power}}$$
2. Volumetric efficiency (η_v) = $\frac{\text{Volume of water actually striking the runner}}{\text{Volume of water supplied to the turbine}}$
3. Overall efficiency (η_o) = $\frac{\text{Power available at the shaft of the turbine}}{\text{Power supplied at the inlet of the turbine}}$
$$= \frac{\text{shaft power}}{\text{water power}} \times \frac{R.P}{R.P} = \frac{S.P}{R.P} \times \frac{R.P}{W.P}$$
$$= \eta_m \times \eta_h$$

Relation between mechanical, volumetric and overall efficiencies are $\boxed{\eta_o = \eta_m \times \eta_h}$

$$W.P = \frac{W \times H}{1000} = \frac{\rho g Q H}{1000}$$

$$\eta_o = \frac{S.P}{W.P} = \frac{P}{\frac{\rho g Q H}{1000}} = \frac{P}{\rho g Q H}$$

Points to be remembered in Pelton wheel:

1. Velocity at inlet $V_1 = C_v \sqrt{2gH}$

$$C_v = 0.98 \text{ (or) } 0.99$$

$$2. U = \phi \sqrt{gH}$$

ϕ = speed ratio = 0.43 to 0.48

3. Angle of deflection of the jet through buckets is taken

as 165° if no angle of deflection is given

$$4. U = \frac{TIDN}{60}$$

N = speed in rpm D = Mean diameter

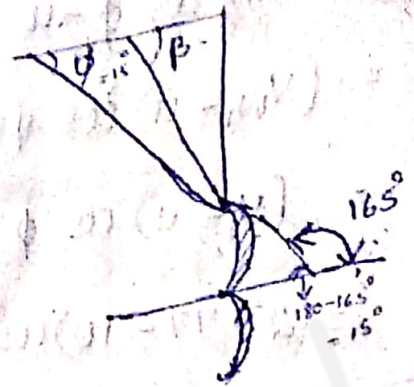
$$5. \text{Jet ratio } (m) = \frac{D}{d} = \frac{\text{Mean diameter}}{\text{Diameter of the nozzle}}$$

For most cases $m = 12$

$$6. \text{No. of buckets } (Z) = 15 + \frac{D}{2d}$$

$$Z = 15 + 0.5 m$$

$$7. \text{No. of jets} = \frac{\text{Total rate of flow through the turbine}}{\text{Rate of flow through a single jet}}$$



Problems

1. A pelton wheel has a mean bucket speed of 10 m/s with a jet of water flowing at the rate of 700 lit/sec under a head of 30 m . The buckets deflect the jet through an angle of 165° . Calculate the power given by water to the runner and the hydraulic efficiency of the turbine. Assume coefficient of velocity as 0.98 .

sol: Given Mean bucket $U = 10 \text{ m/sec}$, discharge $Q = 700 \text{ lit/sec}$
 $Q = 0.7 \text{ m}^3/\text{sec}$

$H = 30 \text{ m}$, Angle of deflection $= 165^\circ$
 $\phi = 180 - 165^\circ \Rightarrow \phi = 15^\circ$

$$C_v = 0.98$$

$$V_{W1} = V_1 = C_v \sqrt{2gH} = 0.98 \sqrt{2 \times 9.81 \times 30}$$

$$V_{W1} = 23.71 \text{ m/s}$$

$$\begin{aligned}
 V_{w2} &= V_{w1} \cos \phi - u \\
 &= (V_{w1} - u) \cos \phi - u \\
 &= (V_1 - u) \cos \phi - u \\
 &= (23.77 - 10) \cos 15^\circ - 10
 \end{aligned}$$

$$V_{w2} = 3.3 \text{ m/sec}$$

$$\text{Power} = \frac{W.D}{1000} \text{ kW}$$

$$= \frac{F \times u}{1000} \text{ kW}$$

$$= \frac{\rho a V_1 [V_{w1} + V_{w2}] \times u}{1000}$$

$$= \rho [V_{w1} + V_{w2}] \times u$$

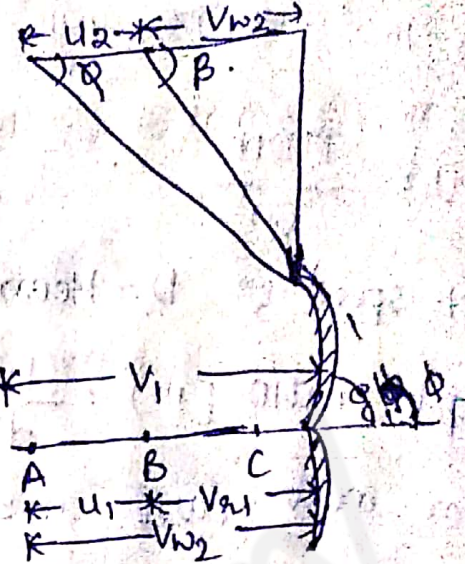
$$= 0.7 [23.77 + 3.3] \times 10$$

$$\text{Power} = 189.49 \text{ kW}$$

$$\eta_h = \frac{2 [V_{w1} + V_{w2}] \times u}{V_1^2} = \frac{2 \times [23.77 + 3.3] \times 10}{(23.77)^2}$$

$$\eta_h = 0.95$$

$$\eta_h = 95\%$$



A Pelton wheel is having a mean bucket diameter of 1 m and is running at 1000 r.p.m. The net head on the pelton wheel is 700 m if the side clearance angle is 15° and discharge through the nozzle is $0.1 \text{ m}^3/\text{sec}$.
 find (i) Power available at the nozzle
 (ii) Hydraulic efficiency of the turbine.

Sol: Given data, $D = 1 \text{ m}$, $N = 1000 \text{ r.p.m.}$, $H = 700 \text{ m}$
 $Q = 0.1 \text{ m}^3/\text{sec}$, $\phi = 15^\circ$

(i) Water power $W.P = \frac{\rho g Q H}{1000} = \frac{9.81 \times 0.1 \times 700}{1000} = 686.7$

(ii) $\eta_h = \frac{Q(V-u)(1+\cos\phi) \times u}{V^2}$

$V_1 = C_v \sqrt{2gH} = 0.98 \sqrt{2 \times 9.81 \times 700} \Rightarrow V_1 = 114.84$

$U = \frac{\pi D N}{60} = \frac{\pi \times 1 \times 1000}{60} \Rightarrow U = 52.35 \text{ m/sec}$

$\eta_h = \frac{Q(114.84 - 52.35)(1 + \cos 15^\circ) \times 52.35}{114.84^2} \times 100$

$\eta_h = 97.1\%$

Two jets strike the buckets of a pelton wheel which is having shaft power 15450 kW. The diameter of each jet is given as 200 mm if the net head on the turbine is 400 m find the over all efficiency of the turbine.
 Take $C_v = 1$

Sol

Given,

Shaft power. S.P = 15450 kW, $d = 200 \text{ mm} = 0.2 \text{ m}$

$H = 400 \text{ m}$,

$C_v = 1$

W.P.T

$$\eta_o = \frac{S.P}{W.P.}$$

$$= \frac{P}{\frac{\rho g Q H}{1000}}$$

$$= \frac{15450}{9.81 \times 5.56 \times 400}$$

$$= 0.708$$

$$= 70.8\%$$

$$Q = a v_1$$

$$= \frac{\pi}{4} d^2 (C_v \sqrt{2gH})$$

$$= \frac{\pi}{4} (0.2)^2 (1 \times \sqrt{2 \times 9.81 \times 400})$$

$$= 2.78 \text{ m}^3/\text{sec}$$

Total discharge from two jets

$$= 2 \times 2.78$$

$$= 5.56 \text{ m}^3/\text{sec}$$

Design of pelton wheel (Impulse)

1. Diameter of the jet (d)

2. Diameter of the wheel (D)

3. Width of buckets (5xd)

4. Depth of buckets (1.2xd)

5. No. of buckets ($z = 1.5 + 0.5m$) $m = \frac{D}{d}$

1. A pelton wheel is to be designed for a head of 60m when running at 200 r.p.m. The pelton wheel develops 95.6475 kW shaft power. The velocity of buckets = 0.45 times the velocity of jet, overall efficiency = 0.85 &

$C_v = 0.98$?

Sol: Given data,

$$N = 200 \text{ r.p.m.}, H = 60 \text{ m}, SP = 95.6475 \text{ kW}$$

$$C_v = 0.98, \eta_o = 0.85$$

$$V_1 = C_v \sqrt{2gH} = 0.98 \sqrt{2 \times 9.81 \times 60} = 33.62 \text{ m/s}$$

$$u = 0.45 \times V_1$$

$$u = 0.45 \times 33.62 = 15.13 \text{ m/sec}$$

$$u = \frac{\pi D N}{60} \Rightarrow D = \frac{15.13 \times 60}{200 \times 3.14} = 1.444 \text{ m}$$

$$W.P = \frac{S.P}{\eta_o} = \frac{95.6475}{0.85} = 119.56 \text{ kW}$$

$$gQH = 119.56 \Rightarrow Q = 0.2 \text{ m}^3/\text{sec}$$

$$Q = aV_1 \Rightarrow a = \frac{Q}{V_1} = \frac{0.2}{33.62}$$

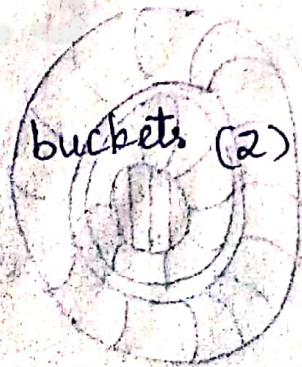
$$\frac{\pi}{4} d^2 = \frac{0.2}{33.62} \Rightarrow d = 0.087 \text{ m} = 87 \text{ mm}$$

$$\text{width of buckets} = 5 \times d = 5 \times 0.087 = 0.435 \text{ m}$$

$$\text{Depth of buckets} = 1.2 \times 0.087 = 0.1044 \text{ m}$$

$$\text{No. of buckets} \left(Z = 15 + 0.5 \frac{D}{d} \right) = 15 + 0.5 \times \frac{1.44}{0.087} = 23.27$$

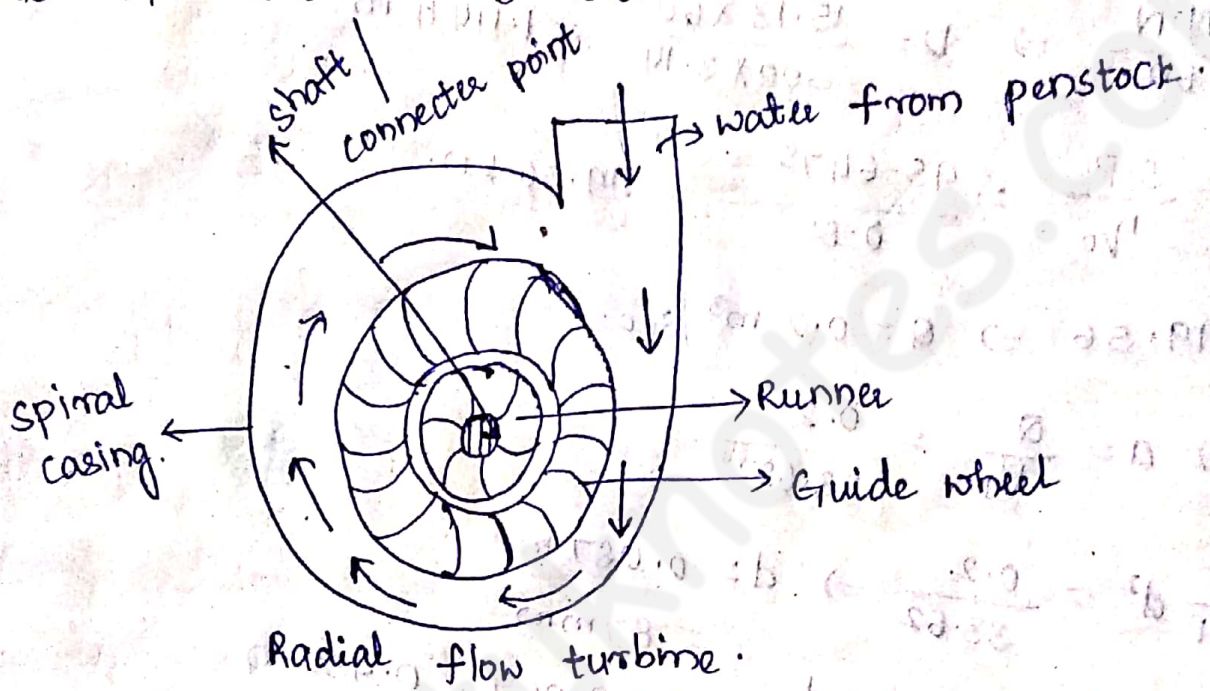
$$\text{No. of buckets (Z)} = 24$$



Reaction radial flow turbine:

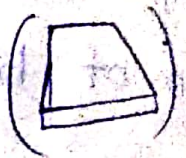
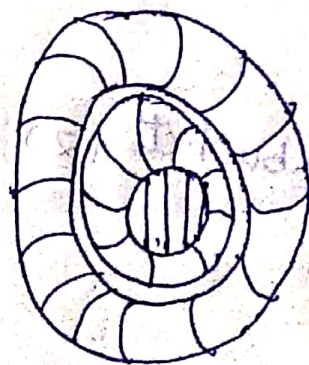
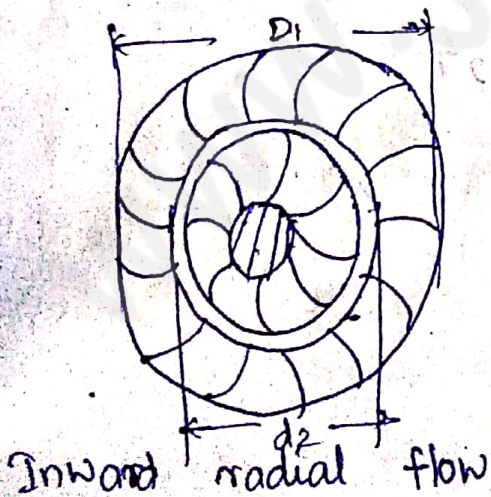
When the water flows in radial direction, the turbine is known as radial flow reaction turbine.

1. If the flow is outward to inward towards the axis of rotation is called inward flow turbine.
2. If the water flow is inward to outward towards the axis of rotation is called outward flow turbine.



Parts of radial flow turbine.

1. Runner
2. Guide wheel
3. spiral casing
4. Draft tube



Work done and efficiency of Inward radial flow turbine:

$$\text{Work done} = F \times u$$

$$= \rho a v_1 [v_{w1} \pm v_{w2}] \times u$$

$$\therefore (u_1 = \frac{\pi D_1 N}{60}, \frac{\pi D_2 N}{60})$$

$$\text{Work done} = \rho a v_1 [v_{w1} \times u_1 + v_{w2} \times u_2]$$

$$\text{Work done / weight} = \frac{1}{g} [v_{w1} u_1 \pm v_{w2} u_2]$$

If $\beta = 90^\circ$, $v_{w2} = 0$

$$= \frac{1}{g} [v_{w1} u_1]$$

The obtained equation is known as Euler's equation or the fundamental equation is known as Hydro dynamic machines. This equation is given by switz scientist

L. Euler

$$\eta_h = \frac{R.P.}{W.P.}$$

$$= \frac{W}{1000g} [v_{w1} \times u_1 \pm v_{w2} u_2]$$

$$\frac{W \times H}{1000}$$

$$= \frac{W}{1000g} [v_{w1} \times u_1 \pm v_{w2} u_2] \times \frac{1000}{W \times H}$$

$$= \frac{[v_{w1} u_1 \pm v_{w2} u_2]}{gH}$$

$\beta = 90^\circ$, $v_{w2} = 0$

$$\boxed{\eta_h = \frac{v_{w1} u_1}{gH}}$$

Definitions:

1. Speed ratio: The speed ratio is defined as

$$\phi = \frac{u_1}{\sqrt{2gH}}$$

u_1 = Tangential velocity

2. Flow ratio: The ratio of the velocity of flow at inlet V_{f1} to the velocity given $\sqrt{2gH}$ is known as flow ratio.

$$\phi \text{ Flow ratio} = \frac{V_{f1}}{\sqrt{2gH}} \quad (H = \text{Head of the turbine})$$

3. Discharge of the turbine:

$$Q = \pi D_1 B_1 \times V_{f1} = \pi D_2 B_2 \times V_{f2}$$

where D_1 = Diameter of the runner at inlet

B_1 = width of the runner at inlet

V_{f1} = velocity of flow at inlet

D_2, B_2, V_{f2} are corresponding values at outlet.

4. Head (H): Head of the turbine.

$$(H) = \frac{P_1}{\rho g} + \frac{V_1^2}{2g}$$

5. Radial discharge: This means the angle made by absolute velocity with the tangent on the wheel is 90° and component of the whirl velocity is 0 (zero). Radial discharge at outlet means $\beta = 90^\circ$.

6. If there is no loss of energy when water flows through the vanes then we have the below equation.

$$H - \frac{V_2^2}{2g} = \frac{1}{g} [V_{w1} u_1 \pm V_{w2} u_2]$$

Problems

1. An inward flow reaction turbine, has external and internal diameters as 1m and 0.5m respectively. The velocity of flow through the runner is constant and is equal to 1.5 m/sec.

Determine (i) Discharge through the runner
(ii) width of the turbine at outlet. If the width of the turbine at inlet is 200 mm.

sol: Given $D_1 = 1\text{ m}$, $D_2 = 0.5\text{ m}$, $B_1 = 200\text{ mm}$

$$V_{f1} = 1.5\text{ m/sec} = V_{f2}$$

$$(i) Q = \pi D_1 B_1 \times V_{f1} = \pi D_2 B_2 \times V_{f2} \Rightarrow \pi \times 1 \times 0.2 \times 1.5 = 0.942$$

$$(ii) Q = \pi D_2 B_2 \times V_{f2} \Rightarrow 0.942 = \pi \times 0.5 \times B_2 \times 1.5 \Rightarrow \boxed{B_2 = 0.39\text{ m}}$$

OUTWARD FLOW REACTION TURBINE:

$$U_1 < U_2$$

$$D_1 < D_2$$

2. The Internal and external diameters of an outward flow reaction turbine are 1m and 2.75 m respectively. The turbine is running at 250 r.p.m and rate of flow of water through the turbine is 5 m³/sec. The width of the runner is constant at inlet and outlet and is equal to 250 mm. The head on the turbine is 150 m neglecting thickness of the vanes and taking discharge radial at outlet. Determine

(i) vane angles at inlet and outlet

(ii) velocity of flow at inlet and outlet

Given data, $D_1 = 2 \text{ m}$, $D_2 = 2.75 \text{ m}$

$Q = 5 \text{ m}^3/\text{sec}$, Speed $N = 250 \text{ rpm}$

$B_1 = B_2 = 250 \text{ mm} = 0.25 \text{ m}$

$H = 150 \text{ m}$

Find out:

$\theta = ?$, $\phi = ?$, $V_{f1} = ?$, $V_{f2} = ?$

$$\tan \theta = \frac{V_{f1}}{V_{w1}} \Rightarrow \theta = \tan^{-1} \left(\frac{V_{f1}}{V_{w1} - u_1} \right)$$

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 2 \times 250}{60} \Rightarrow u_1 = 26.17 \text{ m/s}$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 2.75 \times 250}{60} \Rightarrow u_2 = 35.99 \text{ m/s}$$

$$Q = \pi D_1 B_1 V_{f1}$$

$$\Rightarrow V_{f1} = \frac{Q}{\pi D_1 B_1} = \frac{5}{\pi \times 2 \times 0.25} \Rightarrow \boxed{V_{f1} = 3.18 \text{ m/sec}}$$

$$\Rightarrow V_{f2} = \frac{Q}{\pi D_2 B_2} = \frac{5}{\pi \times 2.75 \times 0.25} \Rightarrow \boxed{V_{f2} = 2.31 \text{ m/s}}$$

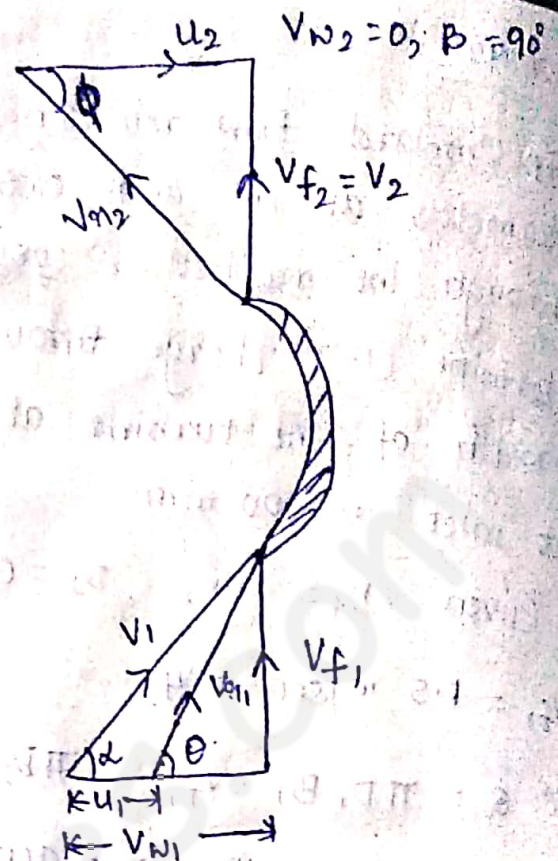
$$H = \frac{V_2^2}{2g} = \frac{1}{g} [V_{w1} - u_1] \quad [2: V_2 = V_{f2}]$$

$$150 = \frac{(2.31)^2}{2 \times 9.81} = \frac{1}{9.81} [V_{w1} \times 26.17]$$

$$\boxed{V_{w1} = 56.12 \text{ m/s}}$$

$$\theta = \tan^{-1} \left(\frac{3.18}{56.12 - 26.17} \right) \Rightarrow \boxed{\theta = 6^\circ 3' 38''}$$

$$\phi = \tan^{-1} \left(\frac{V_{f2}}{u_2} \right) = \tan^{-1} \left(\frac{2.31}{35.99} \right) \Rightarrow \boxed{\phi = 3^\circ 40'}$$



FRANCIS TURBINE:

The inward flow reaction turbine having radial discharge at outlet is known as Francis turbine.

In the modern Francis turbine the water enters the runner of the turbine in radial direction at outlet and leaves in the axial direction at the inlet of the runner. Thus, the Francis turbine is a mixed flow type turbine.

→ Workdone by water on the runner per second will be

$$W.D/sec = \rho Q [v_{w1} u_1]$$

→ Workdone per unit weight of water striking per second.

$$= \frac{1}{g} [v_{w1} u_1]$$

→ Hydraulic efficiency will be given by $\eta_h = \frac{v_{w1} u_1}{gH}$

→ The ratio of width of the wheel to its diameter is given as $n = \frac{B_1}{D_1}$

n varies from 0.10 to 0.40.

→ Flow ratio $= \frac{V_{f1}}{\sqrt{2gH}}$ This ratio varies from 0.15 to 0.30.

→ The speed ratio $= \frac{u_1}{\sqrt{2gH}}$ This ratio varies from 0.6 to 0.9.

→ Hydraulic efficiency $\eta_h = \frac{\text{Total head at inlet} - \text{Hydraulic loss}}{\text{Head at inlet}}$

A Francis turbine with an overall efficiency of 75% is required to produce 148.25 kW power. It is working under a head of 7.68 m. The peripheral velocity equal to $0.26 \sqrt{2gH}$ and the radial velocity of velocity is $0.96 \sqrt{2gH}$. The wheel runs at 150 r.p.m and the hydraulic losses in the turbine are 22% of the available energy. Assuming the radial discharge. Determine

- (i) The guide blade angle.
- (ii) The wheel vane angle at inlet.
- (iii) Diameter of the wheel at inlet.
- (iv) Width of the wheel at inlet.

sol: Note: When hydraulic loss is given like percentage of available energy we have to take it as a value by multiplying with H .

Ex: 22% , 20%
 $0.22H$, $0.20H$

Given, $\eta_o = 75\%$, S.P = 148.25 kW, $H = 7.62$ m, $N = 150$ r.p.m

$$u_1 = 0.26 \sqrt{2gH} = 0.26 \sqrt{2 \times 9.81 \times 7.62} \Rightarrow \boxed{u_1 = 3.17 \text{ m/s}}$$

$$V_{f1} = 0.96 \sqrt{2gH} = 0.96 \sqrt{2 \times 9.81 \times 7.62} \Rightarrow \boxed{V_{f1} = 11.73 \text{ m/s}}$$

Find out:

$\alpha = ?$, $\theta = ?$, $D_1 = ?$, $B_1 = ?$

$$\tan \theta = \frac{V_{f1}}{(V_{w1} - u_1)} \quad \Rightarrow \quad \tan \alpha = \frac{V_{f1}}{V_{w1}}$$

$$Q = \pi D_1 B_1 V_{f1}$$

$$B_1 = \frac{Q}{\pi D_1 V_{f1}}$$

$$u_1 = \frac{\pi D_1 N}{60}, \quad D_1 = \frac{60 u_1}{\pi N}$$

$$\eta_h = \frac{V_{w1} u_1}{gH}$$

$$\eta_h = \frac{T \cdot H - \text{Head loss}}{\text{Head at inlet}}$$

$$\eta_h = \frac{H - 0.22 H}{H}$$

$$= \frac{H(1 - 0.22)}{H} \Rightarrow \eta_h = 0.78 \Rightarrow \boxed{\eta_h = 78\%}$$

$$\eta_h = \frac{V_{w1} u_1}{gH} \Rightarrow 0.78 = \frac{V_{w1} \times 3.17}{9.81 \times 7.62} \Rightarrow \boxed{V_{w1} = 18.39 \text{ m/s}}$$

$$\theta = \tan^{-1} \left[\frac{V_{f1}}{V_{w1} - u_1} \right] = \tan^{-1} \left[\frac{11.78}{18.39 - 3.17} \right] \Rightarrow \boxed{\theta = 37^\circ 44'}$$

$$\alpha = \tan^{-1} \left[\frac{V_{f1}}{V_{w1}} \right] = \tan^{-1} \left[\frac{11.78}{18.39} \right] \Rightarrow \boxed{\alpha = 32^\circ 38'}$$

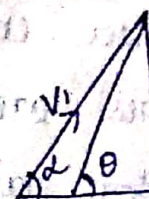
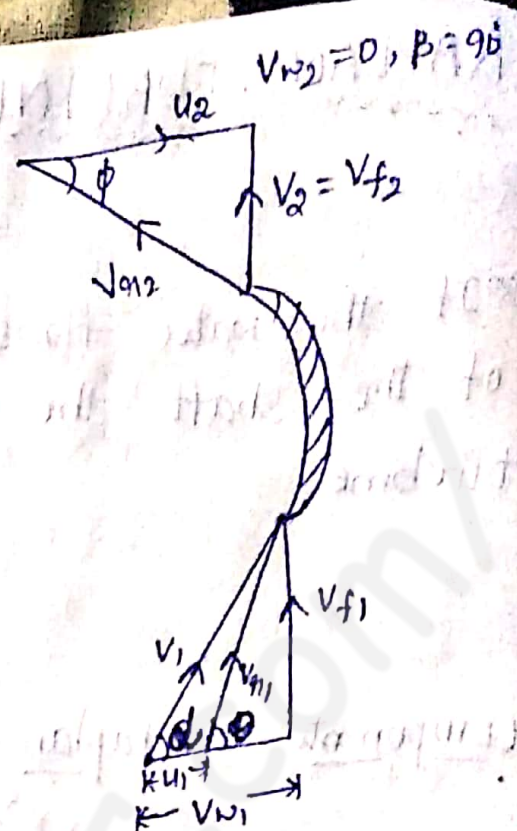
$$D_1 = \frac{60 u_1}{\pi N} = \frac{60 \times 3.17}{\pi \times 150} \Rightarrow \boxed{D_1 = 0.40 \text{ m}}$$

$$Q = a V_1$$

$$a = \frac{\pi}{4} (D_1)^2 = \frac{\pi}{4} (0.40)^2 \Rightarrow a = 0.12$$

$$\boxed{Q = 2.73 \text{ m}^3/\text{s}}$$

$$B_1 = \frac{Q}{\pi D_1 V_{f1}} = \frac{2.73}{\pi \times 0.40 \times 11.73} \Rightarrow \boxed{B = 0.185 \text{ m}}$$



$$\sin \alpha = \frac{V_{f1}}{V_1}$$

$$V_1 = \frac{V_{f1}}{\sin \alpha}$$

$$V_1 = \frac{11.73}{\sin 32^\circ 38'}$$

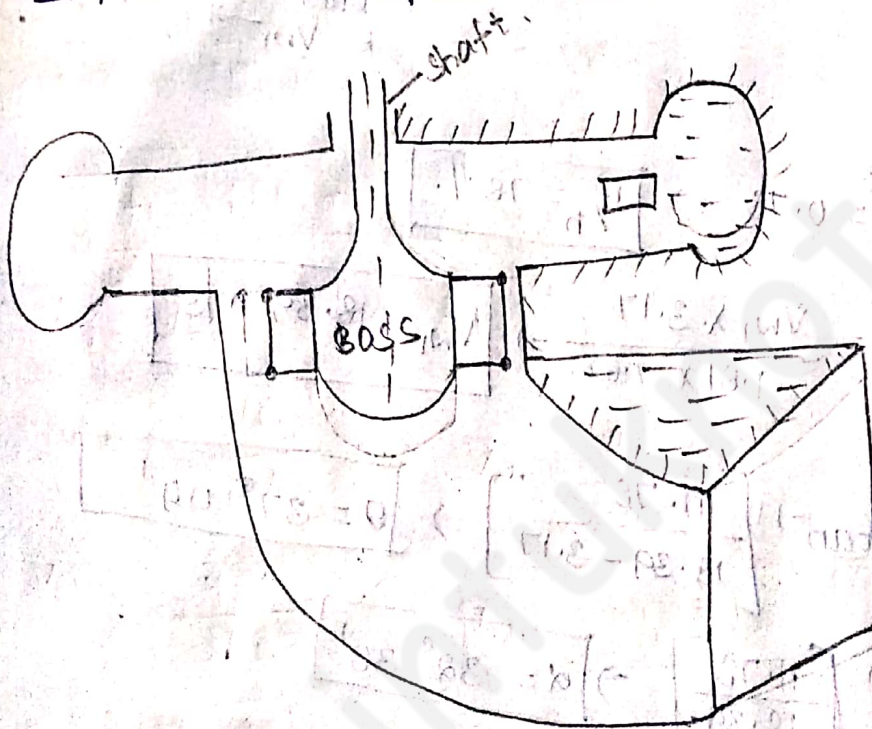
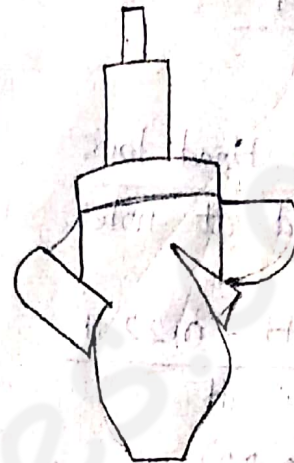
$$\boxed{V_1 = 21.75}$$

KAPLAN TURBINE (OR) AXIAL FLOW REACTION

TURBINE

If the water flows parallel to the axis of rotation of the shaft the turbine is known as axial flow turbine.

Components of Kaplan turbine:



For axial flow reaction turbine the shaft of the turbine is vertical. The lower end of the shaft is made with Hub (or) bass. The vanes are fixed on the hub hence hub acts as the runner for axial flow reaction turbine.

The main parts of the Kaplan turbine are

1. Scroll casing
2. Guide vanes mechanism
3. Hub with vanes
(a) bass with vanes
runner of the turbine

4. Draft tube.

Formulas:

1. Discharge through the runner is obtained as

$$Q = \frac{\pi}{4} (D_o^2 - D_b^2) \times V_{f1}$$

Where D_o = outer diameter of the runner.

D_b = Diameter of the hub, and

V_{f1} = Velocity of flow at inlet

2. The peripheral velocity at inlet and outlet are $u_1 = u_2 = \frac{\pi D_o N}{60}$
velocity of flow at inlet and outlet are equal.

3. Area of flow at inlet = Area of flow at outlet = $\frac{\pi}{4} (D_o^2 - D_b^2)$

1. A Kaplan turbine working under a head of 20 m develops 11772 kW shaft power. The outer diameter of the runner is 3.5 m and hub diameter is 1.75 m. The guide blade angle at the extreme edge of the runner is 35° . The hydraulic and overall efficiencies of the turbines are 88% & 84% respectively. If the velocity of whirl is 0 at outlet, determine

(i) runner vane angles at inlet & outlet at the extreme edge of the runner.

(ii) speed of the runner.

Given $D_o = 3.5$ m, $D_b = 1.75$ m, $\alpha = 35^\circ$, $H = 20$ m

Power of the shaft = 11772 kW

$V_a = 0$

$$Q = \frac{\pi}{4} (D_o^2 - D_b^2) V_{f1}$$

Water power

$$\eta_o = \frac{S.P.}{W.P.} = \frac{11772}{\frac{8984}{1000}}$$

$$0.84 = \frac{11772}{9.81 \times 8 \times 20}$$

$$Q = 71.42 \text{ m}^3/\text{s.}$$

$$Q = \frac{\pi}{4} (D_o^2 - D_b^2) \times V_{f1}$$

$$71.42 = \frac{\pi}{4} [(3.5)^2 - (1.75)^2] \times V_{f1}$$

$$V_{f1} = 9.89 \text{ m/sec}$$

$$V_{f1} = V_{f2} = 9.89 \text{ m/sec.}$$

$$\tan \alpha = \frac{V_{f1}}{V_{w1}} \Rightarrow V_{w1} = \frac{V_{f1}}{\tan \alpha}$$

$$V_{w1} = \frac{9.9}{\tan 35^\circ} \Rightarrow V_{w1} = 14.13 \text{ m/sec}$$

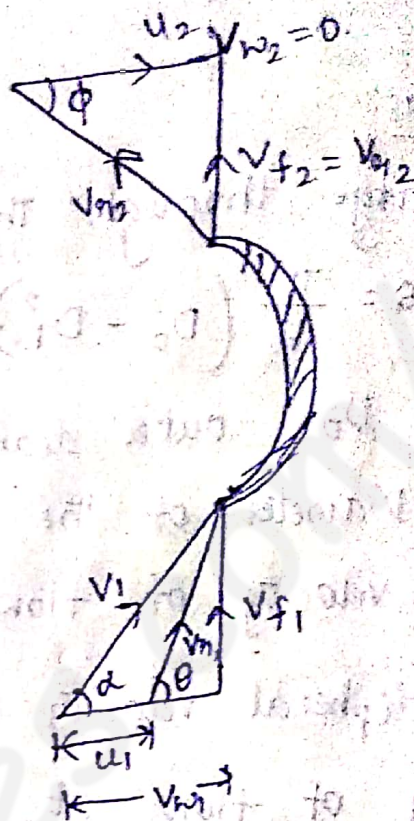
$$\eta_h = \frac{V_{w1} u_1}{g \times H} \Rightarrow 0.88 = \frac{14.13 \times u_1}{9.81 \times 20}$$

$$u_1 = 12.21 \text{ m/sec}$$

$$\tan \theta = \frac{V_{f1}}{V_{w1} - u_1} = \frac{9.9}{14.13 - 12.21}$$

$$\theta = 79.1^\circ$$

$$\phi = \tan^{-1} \left(\frac{V_{f2}}{u_2} \right)$$



$$= \tan^{-1} \left(\frac{9.9}{12.21} \right)$$

$$\phi = 39^\circ 21'$$

$$u_1 = u_2 = \frac{\pi D_o N}{60}$$

$$12.21 = \frac{\pi \times 3.5 \times N}{60}$$

$$N = 66.62 \Rightarrow N = 66$$

$$N = 67 \text{ r.p.m.}$$

Draft tubes:

Types of Draft tubes:

1. conical draft tube



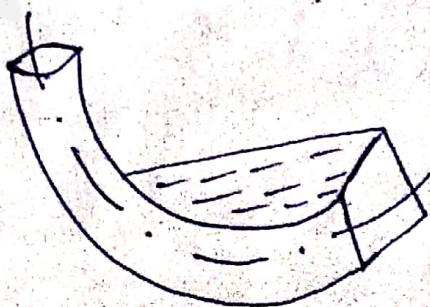
2. simple Elbow draft tube



3. Moody spreading draft tube

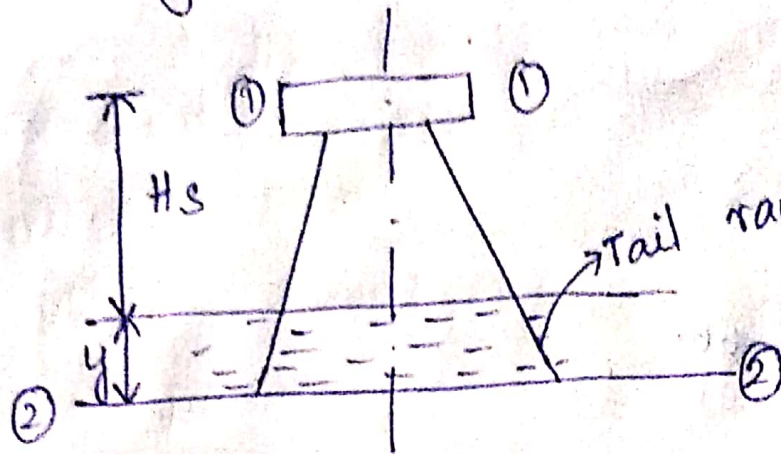


4. Tube with circular inlet and rectangular outlet



Draft tube theory:

Consider a graphical draft tube as shown in figure.



$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + H_s + y = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + 0 + h_f$$

$$\frac{P_2}{\rho g} = \text{Atmospheric pressure} + y$$

$$\frac{P_2}{\rho g} = \frac{P_a}{\rho g} + y$$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + H_s + y = \frac{P_a}{\rho g} + y + \frac{V_2^2}{2g} + h_f$$

$$\boxed{\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + H_s = \frac{P_a}{\rho g} + \frac{V_2^2}{2g} + h_f}$$

Efficiency of the draft tube:

$$\eta_d = \frac{\left[\frac{V_1^2}{2g} - \frac{V_2^2}{2g} \right] - h_f}{\left[\frac{V_1^2}{2g} \right]}$$

A water turbine has a velocity of 6 m/sec at the entrance to the draft tube and the velocity of 1.2 m/sec at the exit. For frictional losses of 0.1 m and a tail water 5 m below the entrance to the draft tube. Find the pressure head at the entrance.

sol: $V_1 = 6 \text{ m/sec}$, $V_2 = 1.2 \text{ m/sec}$, $h_f = 0.1 \text{ m}$, $H_s = 5 \text{ m}$

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + H_s = \frac{P_a}{\rho g} + \frac{V_2^2}{2g} + h_f$$

$$\frac{P_1}{\rho g} = \frac{P_a}{\rho g} + \frac{V_2^2}{2g} + h_f - \left[\frac{V_1^2}{2g} + H_s \right]$$

$$= \frac{P_a}{\rho g} + \frac{(1.2)^2}{2 \times 9.81} + 0.1 - \left[\frac{(6)^2}{2 \times 9.81} + 5 \right]$$

$$= \frac{P_a}{\rho g} + (-6.66)$$

If $\frac{P_a}{\rho g} = 0$ then $\frac{P_1}{\rho g} = 0 - 6.66 = -6.66$ (vacuum pressure)

If $\frac{P_a}{\rho g} = 10$ then $\frac{P_1}{\rho g} = 10 - 6.66 = 3.34$

Specific speed:

It is defined as the speed of the turbine which is identical in shape, geometric dimensions, blade angles, gate opening etc.

Derivation of the specific speed:

$$\eta_o = \frac{\text{S.P.}}{\text{W.P.}} = \frac{P}{\rho g Q H} \times \frac{1000}{1000}$$

$$\Rightarrow \eta_o = \eta_o \times \frac{\rho g Q H}{1000}$$

η_o = overall efficiency.

Q = discharge through the turbine.

$$\Rightarrow P \propto Q \times H.$$

$$u \propto V$$

$$V \propto \sqrt{H}$$

$$u \propto \sqrt{H}$$

$$u = \frac{\pi D N}{60}$$

$$u \propto D N$$

$$\sqrt{H} \propto D$$

$$D \propto \frac{\sqrt{H}}{N}$$

$$\Rightarrow Q = A \times V.$$

$$A \propto B \times D.$$

$$A \propto D \times D.$$

$$A \propto D^2.$$

$$V \propto \sqrt{H}$$

$$Q \propto D^2 \times \sqrt{H}.$$

$$\rightarrow Q \propto \left(\frac{\sqrt{H}}{N} \right)^2 \times \sqrt{H}$$

$$Q \propto \frac{H^{3/2}}{N^2}$$

$$\rightarrow P \propto \left[\frac{H^{3/2}}{N^2} \right] \times H$$

$$P \propto \frac{H^{5/2}}{N^2}$$

$$P = k \cdot \frac{H^{5/2}}{N^2}$$

If $P=1$, $H=1$, $N=N_s$.

$$1 = k \cdot \frac{(1)^{5/2}}{N_s^2}$$

$$N_s^2 = k$$

$$\therefore P = N_s^2 \times \frac{(H)^{5/2}}{N^2}$$

$$\Rightarrow N_s^2 = \frac{N^2 P}{(H)^{5/2}} \Rightarrow N_s = \frac{\sqrt{N^2 P}}{\sqrt{H^{5/2}}}$$

$$N_s = \frac{N \sqrt{P}}{H^{5/4}}$$

13/2020

A turbine is to operate under a head of 25 m at 200 rpm, the discharge is 9 cumics. If the efficiency is 90%. Determine (i) specific speed of the machine.

(ii) power generated (iii) Type of the turbine.

Given data, $H = 25 \text{ m}$, $N = 200 \text{ r.p.m.}$

$$Q = 9 \text{ cumics} = 9 \text{ m}^3/\text{sec.}$$

$$\eta_o = 90\% = 0.90$$

$$N_s = \frac{N \sqrt{P}}{H^{5/4}}$$

$$\eta_o = \frac{S.P}{N.P} = \frac{P}{\frac{\rho g Q H}{1000}} \Rightarrow P = \eta_o \times \rho g Q H = 0.90 \times 9.81 \times 9 \times 25.$$

$$P = 1986.52 \text{ kW}$$

$$N_s = \frac{200 \sqrt{1986.52}}{(25)^{5/4}} \Rightarrow N_s = 139.4 \text{ r.p.m.}$$

$$1 \text{ cumic} = 1 \text{ m}^3/\text{sec}$$

Q.1) As the specific speed lies between 51 & 225.

The turbine is Francis turbine.

2. The turbine develops 9000 kW when running at a speed of 140 r.p.m under a head of 30m. Determine the specific speed of the turbine.

Sol: $N_s = \frac{N \sqrt{P}}{(H)^{5/4}} = \frac{140 \sqrt{9000}}{(30)^{5/4}} \Rightarrow N_s = 189.16 \text{ r.p.m}$

Unit Quantities:

The following are the three important unit quantities which must be studied under a unit head.

(i) unit speed, (ii) Unit power, (iii) Unit discharge.

(i) Unit speed:

$$N_u = \frac{N}{\sqrt{H}}$$

(ii) Unit discharge: $Q_u = \frac{Q}{\sqrt{H}}$

(iii) Unit power $\therefore P_u = \frac{P}{H^{3/2}}$

(i) Unit speed:

$$u \propto v$$

$$v \propto \sqrt{H} \Rightarrow u \propto \sqrt{H}$$

$$u = \frac{\pi D N}{60}$$

$$u \propto N$$

$$N \propto \sqrt{H}$$

$$N = k \sqrt{H}$$

$$\text{If } N=1, N=N_u$$

$$\Rightarrow N_u = k \sqrt{1}$$

$$N_u = k$$

$$\therefore N = N_u \sqrt{H}$$

(ii) Unit discharge:

$$Q = A \times V$$

$$Q \propto V \propto \sqrt{H}$$

$$Q = K \sqrt{H}$$

if $Q = Q_u$, $H = 1$.

$$Q_u = K \sqrt{1}$$

$$Q_u = K$$

$$Q = Q_u \sqrt{H}$$

$$Q_u = \frac{Q}{\sqrt{H}}$$

(iii) Unit power:

$$\eta_o = \frac{P}{\frac{\rho g Q H}{1000}}$$

$$P = \eta_o \times \frac{\rho g Q H}{1000}$$

$$P \propto Q \times H$$

$$P \propto \sqrt{H} \times H$$

$$P \propto H^{3/2}$$

$$P = K \times H^{3/2}$$

if $P = P_u$, $H = 1$.

$$P_u = K \times (1)^{3/2}$$

$$P = P_u \times (H)^{3/2}$$

$$P_u = \frac{P}{H^{3/2}}$$

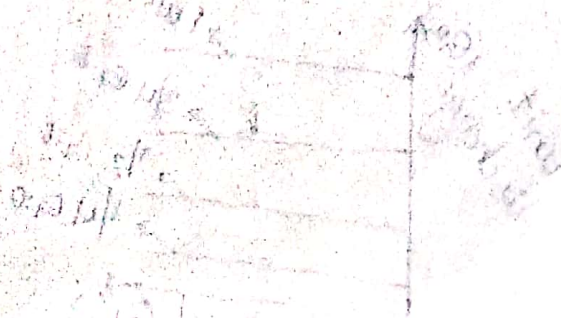
(iii) Unit power:

$$\eta_o = \frac{P}{\frac{\rho g Q H}{1000}}$$

$P =$

1. A turbine develops 9000 kW when running at 10 r.p.m. The head on the turbine is 30 m. If the head on the turbine is reduced to 18 m. Determine the speed and power developed by the turbine.

Sol:



Characteristic curves of the turbines:

characteristic curves are used for ^{knowing} exact behaviour of the turbines under different working conditions.

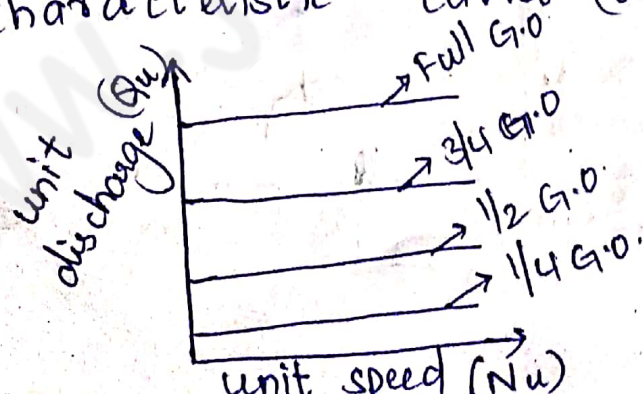
Important parameters which are varied during a test on a turbine:

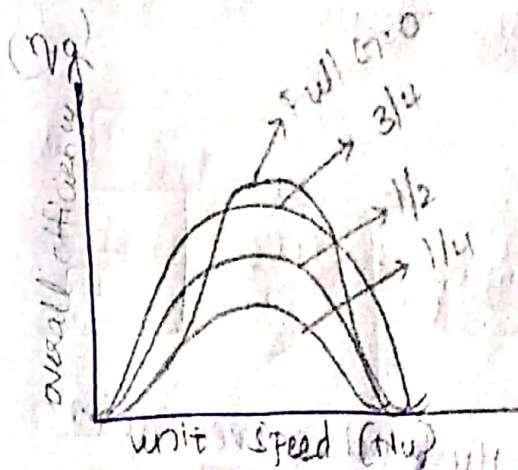
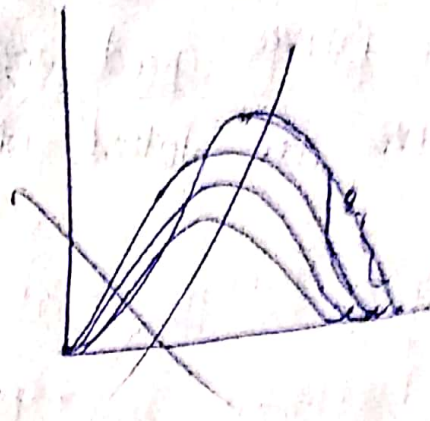
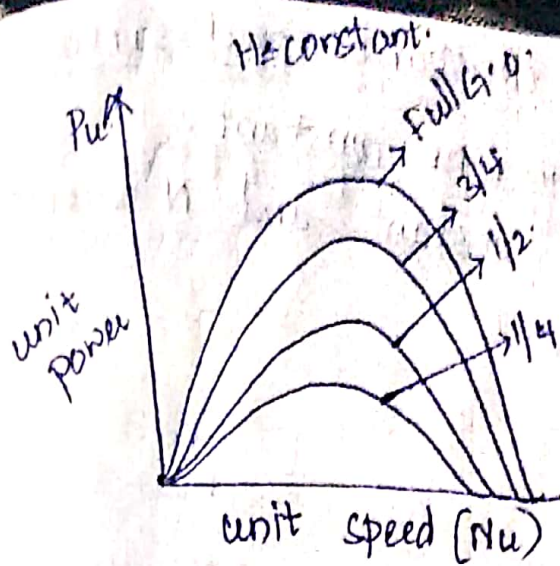
1. Speed (N):
2. Head (H):
3. Discharge (Q):
4. Power (P):
5. Overall efficiency (η_o):
6. Gate opening (G_o):

In these parameters N , H , Q are independent parameters. here H is constant.

Types of characteristic curves:

1. Main characteristic curves (or) constant head curves.

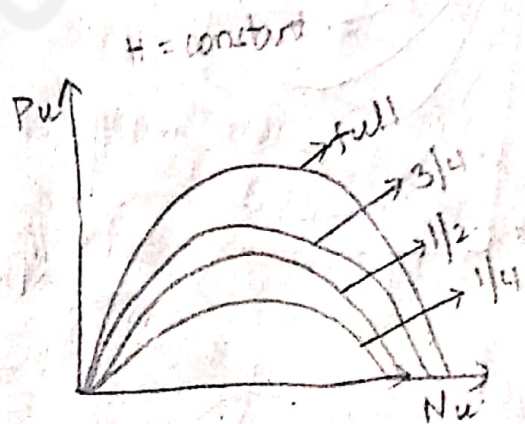
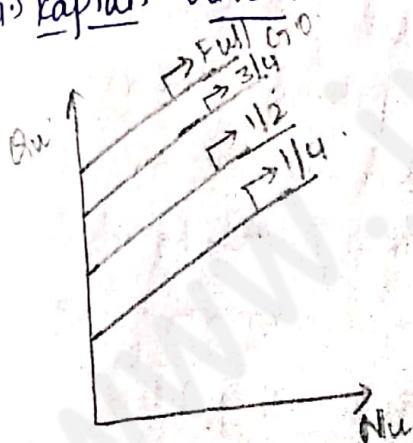




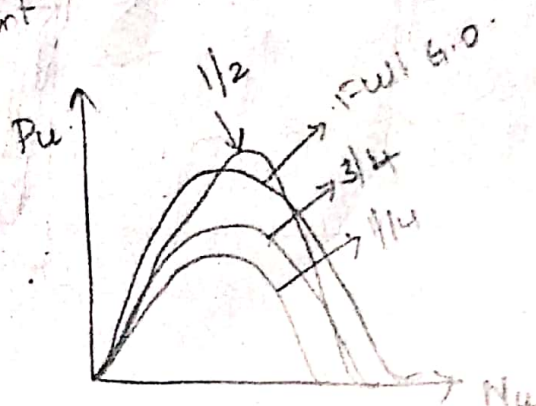
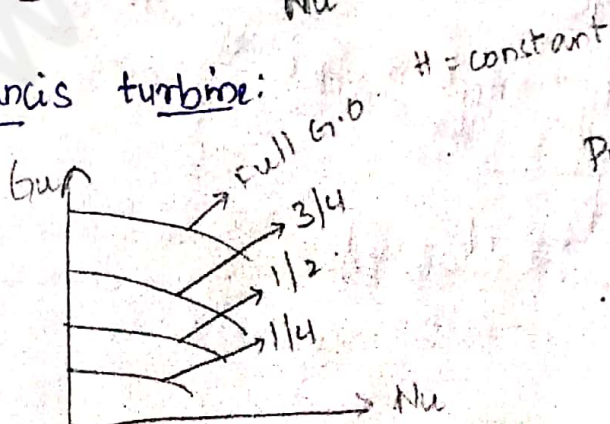
Main characteristic curves of pelton turbine

characteristic curves for reaction turbine:

(a) Kaplan turbine:



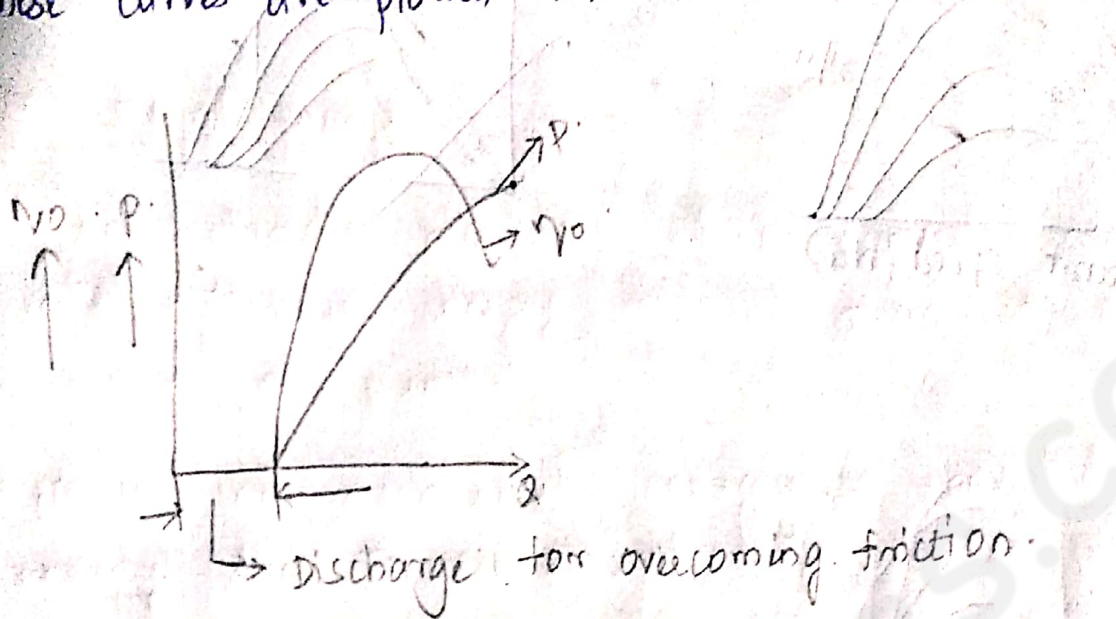
(b) Francis turbine:



2. Operating characteristic curves or constant speed curves:

In these curves speed and head remain constant.

These curves are plotted when H is constant and N is constant.



3. Constant efficiency curves (or) Husel curves (or) Iso-efficiency curves:

